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Anderson Localization and Wave Absorption

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Experimental signatures of classical wave localization in the absence and in the presence of attenuation are analyzed. The different regimes of the attenuation, reflection, and transmission coefficients for the "diffusive" and "localized" regimes are discussed. Apparent contradictory results presented previously by John and Anderson on the renormalization of absorption by localization are reconciled and shown to apply to different situations.

KEY WORDS: Waves and wave propagation; localization in disordered structures.

1. INTRODUCTION

When a wave packet enters an inhomogeneous medium, some of its components suffer scattering events. In such a situation, two and possibly three different regimes occur, according to the distance traveled by the wave from its point of entrance or creation in the medium.[1-3]

(i) At scales smaller than the elastic mean free path l_e, the wave is propagating: its energy travels with an average velocity c and the energy flux $J = cI$ is proportional to the energy density I. In many cases, the energy "transport" of a classical wave in a random medium corresponds to this class, since the mean free path is often found to be larger than realistic sample sizes. This is in contrast to the problem of electronic transport, where the "balistic" regime is rarely encountered.

(ii) At scales larger than l_e, the energy of the wave is diffusive, with a frequency-dependent diffusion coefficient $D_0(\omega) = c(\omega)\, l_e(\omega)/3$. The average velocity of the wave (averaged over a volume $\gg l_e^d$) is zero and the

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energy flux $J = -D_0 \nabla I$ is proportional to the gradient of the energy density. A simple and useful picture is to view the wave as undergoing "random walks" of elementary step length l_e (covered in a "collision" time τ_e with the average velocity c).

(iii) However, in contrast to the transport of molecules in a gas, due to the inherent wave nature of the problem, interference between waves following different diffusing paths occurs. In general, the coherent interference contributions average to zero except in the backward direction, for which two contrapropagating paths correspond to the same phase and therefore interfere constructively for the reflection. For wavelengths $\lambda \ll l_e$, this is the "weak localization" regime, for which the diffusion constant is renormalized to $D = D_0[1 - \gamma(\lambda/l_e)^2]$ in three dimensions,[4] the only case which will be considered here, with γ a number of order one. For larger disorders, l_e decreases and λ/l_e may become of order unity, leading to a breakdown of the perturbation expansion. This regime of strong Anderson localization can be analyzed, among other techniques,[4,5] within a self-consistent diagrammatic approach extending the perturbative weak localization treatment of the corrections to classical diffusive behavior, taking into account the constructing interferences between contrapropagating waves in loops in k space.

Anderson localization is observed or predicted in various electronic systems.[4] Recently, localization has been recognized theoretically as a possible phenomenon for classical waves: e.g., for surface waves in hydrodynamics,[6] in acoustics,[7,8] for electromagnetic waves in plasmas,[9] for third or fourth sound in helium fluid[10] (for review see refs. 2, 3, 11, and 12), and in various experiments proposed or realized[6–8,13–16] mostly for one-dimensional systems or in the weak localization regime. In a previous work,[17] favorable experimental situations which should allow the observation of the strong localization regime in the classical acoustic and electromagnetic wave context were discussed.

In this paper, I attempt to clarify and extend existing work on the experimental signatures of the localization regime. Real systems often present absorption. I therefore focus on the interrelation and competition between wave interferences leading to Anderson localization and the wave absorption characterized by an attenuation length l_a. Other work dealt with this problem, but led to contradictory results, since John[18] predicts an *increase* of the renormalized energy absorption coefficient near the mobility edge, whereas Anderson[19] finds that the absorption (as measure by one minus the reflection coefficient in a semi-infinite slab geometry) *decreases* near the mobility edge. I show that both authors are right and explain that their results apply to different geometries. Using scaling arguments, I confirm the *increase* of the renormalized absorption coefficient in agreement with ref. 18. When the source of the wave energy is placed inside a random medium of size L, this leads to a decrease of the energy leakage outside the cavity of size L as expected. However, in the slab geometry for which a wave is created outside the random medium and tries to propagate through it, the renormalization of the diffusion coefficient tends to prevent the wave from entering significantly in the random absorbing medium and therefore competes with the increase of the renormalized absorption coefficient. It turns out that the decrease in wave penetration wins over the increase of absorption coefficient, leading to a global increase of the reflection coefficient, in agreement with ref. 19.

The presentation is similar to that of Anderson,[19] which I make precise and extend in several aspects.

2. NOTATIONS AND FORMALISM

Following Anderson,[19] I will essentially consider the slab geometry where a wave, with energy density I and energy flux $J = cI$, is incident along the direction $0x$ from the left on a disordered system beginning at $x = 0$ and of total width L. The cavity configuration where the wave source is inside the scattering medium will be briefly addressed, to clarify the role of localization in the renormalization of wave absorption.

In the propagative regime, i.e., at scales $x \leqslant l_e$, $I(x)$ is constant in the absence of dissipation. In the presence of a dissipation described by an attenuation length l_a, the wave energy density decreases as

$$I(x) = I_0 \exp(-x/l_a) \tag{1}$$

This expression will allow us to identify, in the following, the renormalized dissipation length.

The diffusing regime, occurring at scales $x > l_e$, is described by the diffusion equation $\operatorname{div} \mathbf{J} + \partial I/\partial t = 0$, with $J = -D\nabla I$ and appropriate boundary conditions. In the absence of absorption, $\partial I/\partial t = 0$ in the stationary state and one has

$$\operatorname{div}(D\nabla I) = 0, \qquad l_a^{-1} = 0 \tag{2}$$

In the presence of absorption, $\partial I/\partial t = -I/\tau_a$, where $\tau_a = l_a/c$, which gives

$$\operatorname{div}(D\nabla I) - I/\tau_a = 0, \qquad l_a^{-1} \neq 0 \tag{3}$$

Coherent effects leading to localization will be taken into account phenomenologically by introducing in Eq. (2) and (3) a renormalized diffusion coefficient. The analysis will use the results of the self-consistent theory of localization[4,5] in conjunction with scaling ideas applied to the diffusion equation. Coherent effects lead to the introduction of a finite correlation length ξ_+ which is the distance beyond which the diffusion coefficient takes its asymptotic value D. The value of D is smaller than the microscopic D_0 due to coherent backscattering interferences. For stronger disorder ($l_e < l_c$, where $l_c \approx \lambda$ is the localization threshold), one enters into the localization regime and the asymptotic value of the diffusion coefficient vanishes. The typical length scale over which D goes to zero is the so-called localization length, denoted ξ_-.

In the self-consistent theory of localization[4,5] the diffusion coefficient is expressed in terms of ξ_+ (for $l_e > l_c$) or ξ_- (for $l_e < l_c$) as follows:

$$D = D_0 \left[1 - (l_c^2/l_e) \int_0^{1/l_e} q^2 \, dq/(q^2 + \xi_\pm^2) \right] \tag{4}$$

For $l_e > l_c$, there is a simple scaling law relating D to the correlation length ξ_+,

$$D = D_0 l_e / \xi_+ \tag{5}$$

Equation (5) obtained from Eq. (4) is thus consistent with a usual scaling assumption.[20] The localization length ξ_- is, in the self-consistent theory of localization, the infrared cutoff ensuring the nonnegativity of D for $l_e < l_c$. It essentially suppresses the contribution to the renormalized diffusion coefficient of loops larger than ξ_-, thus implying that at scales larger than ξ_-, $D = 0$. It is given by

$$1 - (l_c/l_e)^2 = (l_e/\xi_-) \tan^{-1}(\xi_-/l_e)$$

Both ξ_+ and ξ_- show critical behavior in the vicinity of the critical point $l_e = l_c$:

$$\begin{aligned} \xi_+ &\approx l_e\{(l_e - l_c)/l_e\}^{-\nu} \\ \xi_- &\approx l_e\{(l_c - l_e)/l_e\}^{-\nu} \end{aligned} \tag{6}$$

with $\nu \approx 1/(d-2)$.[2,4,5,20] Note that Eq. (5) is valid for large systems of size $L > \xi_+$. When $l_e \to l_c^+$, $\xi_+ \to +\infty$. In the critical region, L is bound to become smaller than ξ_+ and one is confronted with finite-size effects. Within the scaling theory of localization,[20] a scaling assumption gives for the conductance of electronic systems $g(L) \sim D(L) L^{d-2}$. At (or near) the transition point, $g(L) = g_c = \text{const}$ independent of L and therefore $D(L)$

scales like L^{2-d}, which gives, for $d=3$, a length-dependent diffusion coefficient scaling as

$$D(x) \approx D_0 l_e/x \tag{7}$$

This scaling (7) can also be viewed as stemming from a finite-size scaling ansatz: $D = D_0(l_e/\xi_+) f(L/\xi_+)$. Within the self-consistent theory of localization, it results from the appearance of an infrared cutoff L^{-1} in the integration over q performed in Eq. (4). In the limit where $\xi_+ \to +\infty$, the finite-size effect suppresses long paths and therefore changes the renormalization of D to a finite value. Supposing a power-law dependence of $f(z)$ for small z, this recovers Eq. (7). This scaling is also in agreement with a renormalization group treatment using a field-theoretic formalism[18] and diagrammatic techniques.[5]

Equations (2), (3), and (5)–(7) constitute our starting point for discussing the different following regimes.

3. ATTENUATION, TRANSMISSION, AND REFLECTION COEFFICIENTS

3.1. Diffusive Regime ($L > l_e$, $l_c \ll l_e$)

In the absence of dissipation, it is easy to solve Eq. (2) with $D = D_0$ independant of x with propagating condition for $x < 0$ and $x > L$[1,3] for the transmission coefficient

$$T_0 = D_0/cL \tag{8}$$

The reflection coefficient is of course $R = 1 - T_0$.

In the presence of a small dissipation (l_a finite but large), the result is very different from Eq. (8) and is obtained from Eq. (3):

$$T \approx \exp\{-L/L_a\} \qquad \text{for} \quad L > L_a = (l_a l_e)^{1/2} \tag{9}$$

which defines a new dissipation length "renormalized" by the diffusive nature of the wave energy transport. This result can be interpreted intuitively as follows. The transport of the energy can be described in terms of a sum over random walks W of different length Γ whose average "true" length is $\langle \Gamma \rangle \approx cL^2/D \approx L^2/l_e$ ($\langle \Gamma \rangle /c$ is the average time taken by the diffusive wave to cross the distance L). Here, I neglect the difference between D/c and l_e. One can estimate T by a sum over paths of length Γ, each weighted by the corresponding dissipation factor $\exp(-\Gamma/l_a)$:

$$T \approx \int d\Gamma\, P(\Gamma) \exp(-\Gamma/l_a) \tag{10}$$

where $P(\Gamma)$ is the fraction of the paths having the real length Γ for a gyration radius L: $P(\Gamma) \approx \Gamma^{-d/2} \exp(-L^2/2l_e\Gamma)$. Putting this expression in the integral and computing it within a saddle point approximation for the factor $-F = L^2/2l_e\Gamma + \Gamma/l_a + (d/2)\log \Gamma/l_e$ in the exponential yields the value of Γ at the saddle point:

$$\begin{aligned} \Gamma_{\text{saddle}} &\approx L(l_a/l_e)^{1/2} \qquad \text{for} \quad L > L_a \\ &\approx L^2/l_e \qquad \text{for} \quad L < L_a \end{aligned} \tag{11}$$

Note that the expression for Γ_{saddle} is not changed when taking into account the distortion of the probability distribution $P(\Gamma)$ due to the constraint that the paths must reach the "interface" $x = L$ for the first time without previous crossing. One thus recovers Eq. (9) for $L > L_a$ whereas

$$T \approx \exp(-L^2/L_a^2) \qquad \text{for} \quad L < L_a$$

Equation (11) shows that the transmission of the wave energy is controlled by rare but "efficient" paths whose real lengths scale as L instead of L^2 for typical random walks.

At scales $x < L_a$, Eq. (3) simplifies to Eq. (2), stating that the medium is nonabsorbent up to the thickness L_a. Following the argument of ref. 19, one estimates the reflection coefficient as that of an effective nonabsorbing system of size L_a,

$$R = 1 - T_0(L_a) = 1 - (l_e/l_a)^{1/2} \tag{12}$$

Exact results using Boltzmann-type radiation transfer equation confirm this well-known result.[1, 22] Note that the effective attenuation length L_a is the mean distance covered by the diffusive wave energy on a time equal to the dissipation time τ_a:

$$D(L_a)\tau_a \approx L_a^2 \tag{13}$$

3.2. Critical Transport ($L > l_e$, $l_c \approx l_e$)

3.2.1. Case $L > \xi_+$. In the absence of dissipation, one can use the expression (8) for T_0 with the correct value of D given by Eq. (5). Using the scaling law (6) for ξ_+, one obtains the expression for the transmission factor

$$T_0^c \approx (l_e - l_c)/L \tag{14}$$

This is in agreement with ref. 19 after correcting a misprint.

In the presence of dissipation, since D given by Eq. (5) is a constant at the scale L, the discussion follows that of Section 3.–1 and the results can be transcribed by replacing l_e by $l_e^2/\xi_+ \approx l_e - l_c$, that is, L_a by

$$L_a^c \approx [l_a(l_e - l_c)]^{1/2} = [l_a l_e]^{1/2}(l_e/\xi_+)^{1/2} \tag{15}$$

L_a^c can also be obtained directly from an argument similar to that of Eq. (13): $D(\xi_+)\tau_a \approx L_a^{c2}$. I have made use of Eq. (13), which is the basis of the relationship between attenuation and the scaling of the diffusion coefficient.

In sum, we obtain

$$T \approx \exp[-(L/L_a^c)^2] \qquad \text{for} \quad \xi_+ < L < L_a^c \tag{16}$$

$$T \approx \exp(-L/L_a^c) \qquad \text{for} \quad \xi_+ < L_a^c < L \tag{17}$$

valid not too near the critical point ($\xi_+ < L_a^c$). By comparison with Eq. (1), Eq. (17) defines the renormalized dissipation length L_a^c. Its scaling (15) has been obtained previously by John using a field-theoretic formulation,[18] which now takes a straightforward physical meaning: a dissipation length $(l_a D_0/c)^{1/2} \approx (l_a l_e)^{1/2}$ is the result of the random walk trajectory followed by the wave packets, as discussed in Section 3.1. Now, changing D_0 into D leads to Eq. (15), which contains both the effect of the diffusion and that of the coherent effect of localization. It is remarkable that the knowledge of the scaling of the diffusion coefficient enables one to obtain the form of the renormalization of the attenuation $\alpha \approx (L_a^c)^{-1}$, which turns out to be intimately coupled to it.

The reflection coefficient R is obtained as before by putting the value of L_a^c in the expression (14) for T_0^c, which yield[19]

$$R \approx 1 - T_0^c(L_a^c) \approx 1 - [(l_e - l_c)/l_a]^{1/2}$$

3.2.2. Case $L < \xi_+$. In the absence of dissipation, one has to solve Eq. (2) with a scale-dependent diffusion coefficient given by Eq. (7). The result is[19]

$$T \approx (l_e/L)^2 \tag{18}$$

This result is recovered simply from T_0 [Eq. (8)] using the finite-size scaling $D \approx D_0 l_e/L$. This L^{-2} power law is the signature of the localization critical region in the absence of dissipation.

In the presence of dissipation, we have to solve Eq. (3) with a space-dependent diffusion coefficient given by Eq. (7). For large x, its leading form can be rearranged under the form of an Airy equation: $d^2I/dX^2 = XI$, where $X = x/L_a$ and $L_a = (l_e^2 l_a)^{1/2}$. One also finds L_a from Eq. (13) with

$D(L_a) = D_0 l_e/L_a$. The asymptotic expansion of I and therefore of the transmission coefficient T is[21]

$$T \approx (L/L_a)^{-1/4} \exp[-(2/3)(L/L_a)^{3/2}] \tag{19}$$

One expects from an argument similar to that of Section 3.1 that this law is valid only for $L > L_a$. For $L < L_a$, one expects

$$T \approx \exp[-(L/L_a)^3] \tag{20}$$

As before, the reflection coefficient is given by $R = 1 - T_0(L_a)$, where T_0 is given by Eq. (8), which yields[19]

$$R = 1 - (l_e/l_a)^{2/3} \tag{21}$$

Comparing this result (21) with the classical diffusion case given by Eq. (12), we recover the result of Anderson,[19] from which one concludes that absorption (as measured by one minus the reflection coefficient in a semi-infinite slab geometry) *decreases* near the mobility edge. Less energy is absorbed in the medium in the localization regime. As an illustration, take $l_e/l_a \sim 10^{-3}$. This gives a total dissipation $1 - R \approx 3 \times 10^{-2}$ from Eq. (12) and a smaller $1 - R \approx 10^{-2}$ from Eq. (21). This is in contrast with the reduced dissipation length L_a^c given by Eq. (15), which implies a larger attenuation.

3.3. Renormalized Absorption versus Renormalized Diffusion Coefficient

Since the contradictory results of John[18] [Eq. (15)] and Anderson[19] [Eq. (21)] can both be understood within our framework, we are now in position to unravel the contradiction.

3.3.1. Competition between Renormalized Disssipation and Diffusion Coefficient. Localization increases the absorption, i.e., decreases l_a to L_a^c, but it also decreases the diffusion coefficient D according to Eq. (5). As a consequence, the effective penetration of the wave is decreased in the absence of absorption, since the transmission coefficient goes from an l_e/L to an $(l_e/L)^2$ dependence. The renormalization of the diffusion coefficient (5) tends to prevent the wave from entering significantly in the random absorbing medium and therefore competes with the increase of the renormalized absorption coefficient. It turns out that the decrease in wave penetration wins over the increase of absorption coefficient, leading to a global increase of the reflection coefficient, in agreement with ref. 19. This effect can be viewed as a kind of increased impedance mismatch due to localization which prevents the wave from penetrating the absorbing medium and therefore which "protects" it from being absorbed.

3.3.2. Case of the "Cavity" Geometry. The competition between "penetration in the medium" and attenuation can be made clearer by the analysis of the cavity configuration where the wave is created inside the diffusing medium. In this case, we can study a spherically symmetric geometry, where r represents the distance from the origin where the source emits isotropically a power P_s. Then, Eq. (3) is replaced by $r^{-2}\,\partial\{r^2 D(r)\,\partial I/\partial r\}/\partial r - I/\tau_a = 0$. If $D(r)$ is a constant, it simplifies to $r^{-1}\,\partial^2(rI)/\partial r - I/D\tau_a = 0$.

In the diffusing regime ($l_e \gg l_c$), for $l_a^{-1} = 0$, the wave power flux at a distance r from the source is $J(r) \approx P_s/4\pi r^2$, leading to an outgoing power $4\pi r^2 J(r) = P_s$ corresponding to the conservation of the energy. For $l_a^{-1} \neq 0$, $J(r) \approx P_s r^{-2} e^{-r/L_a}$, leading to an outgoing power at distance r of the order of $P_s e^{-r/L_a}$ with $L_a = (l_e l_a)^{1/2}$ given by Eq. (9).

In the critical regime, we can solve approximately the diffusion equation in the presence of absorption and obtain the outgoing power, which is of the order of[21]

$$T \approx P_s(r/L_a)^{3/2} \exp\{-(2/3)(r/L_a)^{3/2}\} \qquad \text{where} \quad L_a = (l_e^2 l_a)^{1/3} \tag{22}$$

For $r \gg L_a$, only a vanishingly small proportion of P_s leaks out of the cavity of size r; the rest of the power is absorbed by the medium.

The comparison between the cavity and the slab geometry shows that when the wave is completely inside the medium, the localization regime leads indeed to an impressive increase of the absorption, in agreement with ref. 18. However, when the wave must enter the medium as in the slab geometry, the renormalization of the absorption is more than compensated by the reduced penetration of the wave.

3.4. Localization Regime

For $l_e < l_c$, D vanishes at large scales. The transport of the energy is slower than diffusive. From very general arguments,[2,4,20] essentially reasoning connecting the transmission coefficient in energy to the conductance by the Landauer formula[1,4] $G = (2e^2/h)\,T/(1-T)$, one expects a transmission T *in the absence of any dissipation* decaying exponentially with L as

$$T_{\text{loc}} \approx (l_e/L)^2 \exp(-L/\xi_-) \qquad \text{for} \quad L < \xi_- \tag{23}$$

$$\approx [(l_c - l_e)/L] \exp(-L/\xi_-) \qquad \text{for} \quad L > \xi_- \tag{24}$$

where the localization length ξ_- is given by Eq. (6) near the critical point $l_e = l_c$ ($\approx \lambda$ according to the Ioffe–Regel criterion). The $(l_e/L)^2$ prefactor in Eq. (23) ensures that T is continuous at $l_e = l_c$.

One could expect that, for a finite system of size L, the diffusion coefficient should scale like $D(L) \approx D_0 e^{-L/\xi_-}$ for large L, as suggested by the scaling of the conductance $g = g_0 e^{-L/\xi_-}$. But in the scaling theory,[20] all the subtlety comes from the fact that g still has a meaning in the localization regime, as does the conductivity σ if one believes in the relation $g = \sigma L^{d-2}$. But D is related to σ via the Einstein relation $\sigma = ne^2 D$ (n is the density of carriers), which is wrong in the localized regime.[4],[2] We can see the difficulty directly on the propagator $P(r, t)$, which is proportional to the probability for a wave packet to be at position r at time t starting from the origin at time 0:

$$P(r, t) \sim \exp(-r^2/Dt) \qquad \text{for} \quad r < \xi_-$$
$$\text{or} \quad P(k, \omega) \sim (k^2 - i\omega/D)^{-1} \qquad \text{for} \quad k > \xi_-^{-1} \tag{25}$$
$$P(r, t) \sim \exp(-r/\xi_-) \qquad \text{for} \quad r > \xi_-$$
$$\text{or} \quad P(k, \omega) \sim (k^2 + \xi_-^{-2})^{-1} \qquad \text{for} \quad k < \xi_-^{-1} \tag{26}$$

where $P(k, \omega)$ is the spatiotemporal Fourier transform of $P(r, t)$. Equation (25) shows that the mean square displacement $\langle r^2 \rangle$ scales as Dt for $t < \xi_-^2/D$, whereas it saturates at ξ_-^2 at larger times as seen from Eq. (26) and known from a rigorous result.[23] This behavior is incompatible with the scaling one would obtain from the assumption $D(L) \approx D_0 e^{-L/\xi_-}$, since this would lead to $r \sim \xi_- \log t$ [with the help of the usual relationship $r^2 \sim D(r)t$] in obvious contradiction with the rigorous result.[23] Intuitively, the finiteness of $\langle r^2 \rangle$ comes from the fact that the localized modes which are visited by the initial wave packet prepared around the origin are coupled only exponentially to the wave packet and this exponentially small coupling enforces a finite energy mean square displacement.

An idea of the scaling of D in this regime may come from the classical definition

$$D = \lim_{t \to +\infty} (2\,dt)^{-1} \int d^d r\, (r - r')^2\, P(r, r', t) \tag{27}$$

Using the form (25) and (26) for $P(r, r', t)$ yields

$$D \sim D_0 t_-/t \qquad \text{for} \quad t > t_- \sim \xi_-^2/D_0 \tag{28}$$

[2] I am grateful to a referee for a judicious remark on this point.

Expression (28) reconciles the existence of a scaling for D with the rigorous result.[23] It shows that the above difficulty comes from a bad choice of the scaling variable, which must be the time instead of the space position. We can now use this scaling for the discussion of the transmission and reflection in the presence of dissipation characterized by a dissipation time $\tau_a = l_a/c$:

For $\tau_a < t_c$, the dissipation occurs before the true strong localization regime and the discussion is similar to the one in the critical regime for $l_e \geqslant l_c$ of Section 3.2.

For $\tau_a > t_c$, we enter the true strong localization regime, which is very difficult to analyze quantitatively.

4. CONCLUSION

The experimental signature of classical wave Anderson localization has been discussed with particular attention to the competition between localization and dissipation. The role of dissipation has been discussed previous[17] in relationship with the width of the "external" and "internal" resonances as a mechanism which prevents efficient scattering and therefore the attainment of the strong localization regime. Bulk dissipation through damping of the wave also plays a crucial role in an experimental setup and a strong attenuation makes the observation of localization difficult. However, the localization regime reveals itself in a strong increase of dissipation in the spherical cavity geometry but in an increase of the reflection in a slab geometry. These results suggest performing experiments involving both geometries in order to obtain a clear-cut experimental signature of Anderson localization. The cavity geometry could be obtained with the use of optical fibers as light sources inside the disordered medium. I hope that the present paper will be useful for analyzing future experiments on the still elusive localization in the classical wave context in three dimensions. These results could also be of practical interest, for example, in the conception of new absorbing or reflecting materials.

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